The Unfinished Manuscript

Aldo Belleni-Morante started to write this book in February 2008 giving two provisional titles: *Semigroups and Evaluation Equations in Locally Convex Spaces*: *An Introduction* or *Applied Semigroups in Locally Convex Spaces* and, he seemed on hurry for finishing it.
He decided to share his scientific viewpoint with the Scottish colleagues Prof. Adam C. McBride (AMB) and Dr Wilson Lamb (WL) from the Strathclyde University. He fully desired this collaboration as a consequence of some previous scientific works undertaken since 2006 at the Strathclyde University along his appointment as Permanent Visiting Professor. He also considered the very early conception of this book since 2005 when he spent his latest sabbatical year in Glasgow and further in 2007 when Adam McBride came to Florence to work on this. But not much work was done at that time.
To this end, Aldo started happily on his own research work to write the book and he completed his first part in 2008. Unfortunately, the first health problems arisen and this book stayed unfinished in this first draft writing.
After Aldo passed away, for several years I was expecting to someone to proceed with the finishing of the book. But, for some reason, this did not happen. Therefore, as I knew how Aldo's this last writing was so valuable to him and how much scientific importance he had put on this, I really desired to explore firstly the ways for a scientific publication, with no result.
But only in recent time I have been increasingly convinced for the publication of the handwritten and unfinished version of this book. The premise is that scientific contents of this book are more durable than anything else and should therefore take priority in our times arguments that may be not difficult to be recognised even in a such a draft version.
For me, as wife but also as architect and professor, the value of this unfinished book, apart its truly scientific value, is undoubtedly on the beautiful compact handwriting, the uniqueness of this extraordinary writing with the pencil, the cleanness of the pages well numbered together with the clearness of the concise and prominent scientific concepts.
For years I loved Aldo's handwriting on large square knitted notebooks. He was a prodigious writer with pages and pages of calculations and theorems including beautiful scientific hand drawings. I always, admired the density of his ´scientific mind´ and all suggestive forms of his academic expression.
The ´simplicity´ of his mind to solve ´easily´ such ´complexity´ of scientific problems and theorems

it was constantly a good training to me too for my personal and academic growth as well. The publication of this book to me is a restorative approach to my husband's memory in putting forward a positive contribution of thinking in the discipline. I think that this unfinished handwriting draft edition can also mark an important milestone in doing research. For sure, can re-frame the scientific debate on the topics under consideration.

Finally, I am sure that Aldo's scientific work enriched the academic live of many researchers in Italy and in abroad and his scientific doing was always forward with the times.

Surely, I am feeling to underline that Professor Aldo Belleni-Morante and his scientific contribution constantly advocated for a fundamental expression within the discipline while his everyday living gave so much 'lightness' to this highly complex thing we call the life.

Aldo was a scientist and maestro of life and he donated to me a lot of happiness, love and scientific awareness.

Dimitra Diana Babalis Belleni-Morante

Aldo Belleni-Morante
Professor and Scientist

Aldo Belleni-Morante (January 22nd, 1938 in Florence - June 20th, 2009 in Florence) was an Italian mathematician, Professor of Mathematical Physics at the University of Florence.
He got a Degree in Physics in 1961 and became a Full Professor being only 33 years old at the School of Science of the University of Bari. In 1974, he moved to Florence, in the School of Engineering, where he became Dean of the Board of Professors in 1996. Since 2000, he was Coordinator of the PhD Program in *Civil and Environmental Engineering* of the University of Florence and in 2005 Director of the PhD Program of the School of Engineering.
His research and teaching experiences in prestigious universities were significant and fruitful; he was *NATO Fellow* in the Department of Physics of the University of Illinois in Urbana-Champaign, USA, (1968-1969); *Senior Visiting Fellow* of the Science Research Council of Great Britain at the Mathematical Institute of the University of Oxford, England, (1976); *Visiting Professor* at the University of KwaZulu-Natal, Durban, South Africa, (1994). Since 1995 he has been appointed as *Permanent Visiting Professor* at the Department of Mathematics and Statistics of Strathclyde University, Glasgow, Scotland, UK.
The research activity of Belleni (as all friends and colleagues used to call him) was variegate and innovative. His first work dates back to 1961, published on the *Rivista di Matematica* of the University of Parma, "On a problem of neutron diffusion". Other papers in Italian followed. They dealt with problems related to heat transfer in nuclear reactors. His first paper in English was "Solution of the nuclear reactor kinetics equations by a method of successive approximations" and dates back to 1963. It appeared in the *Journal of Nuclear Energy*. Subsequent works must be added always appeared on *J. Nuclear Energy*, *Energia Nucleare*, and *Nukleonik* in the years 1965-66.
His first results in mechanics date back to 1968, published on *Meccanica*, the international journal of the Italian Association of Theoretical and Applied Mechanics. The title was "On the principle of gyroscopic effect for self-excited rigid bodies". It is a work of classical mechanics, but already it highlights the features of Belleni's style: rigor, simplicity, and innovation.
Since then, Belleni carried out a dynamic activity and he was very productive. After getting the Chair of Mathematical Physics, he spent three years (from 1971 to 1974) at the Institute of Rational Mechanics in the University of Bari. It was then that he started fruitful collaborations with peers and young students. The scientific collaborations were numerous: we record 25 co-authors, that

demonstrates how great was his generosity and his desire to spread knowledge, methods, and theories.
Belleni came back to Florence in the academic year 1974-1975. Since then he always held the Course of Rational Mechanics within the undergraduate program in Electronic Engineering; then in Mechanical Engineering and in Civil Engineering. In those years, he also held the Course of Advanced Mathematics Applications, *(Applicazioni di Matematiche Superiori)* at the School of Mathematical, Physical and Natural Sciences, and other various courses in the PhD programs of Mathematics and Civil and Environmental Engineering of the University of Florence.

Belleni's initial research interest was addressed towards the physics of nuclear reactors: his contributions based on physical-mathematical aspects were truly remarkable.
Therefore, he had a great capacity to create mathematical models for phenomena in different areas of physics. Scrolling through the list of his publications and consulting *MathReview* it is noted that, after his first interest in classical topics of mechanics, he addressed his attention towards kinetic theory, neutron transport, thermodynamics, vehicular traffic, the theory of populations, astronomy and astrophysics, spatial planning, ecology.
A more precise list of his research areas includes: astronomy and astrophysics, biology, classical thermodynamics and heat transport, fluid mechanics, solid mechanics, statistical mechanics and structure of matter, numerical analysis, operator theory, ordinary differential equations, partial differential equations. The list involves not only applied topics but also issues on pure mathematics. Since 1969, in fact, Belleni began to study the theory of semigroups of operators, until becoming a world-renowned expert.
Starting from the notes of the Advanced Theoretical Mechanics lectures and from different editions of his handouts, he wrote his first book on the theory of operator semigroups with a particular view on results of existence and uniqueness in problems of mathematical physics, entitled: *Applied Semigroups and Evolution Equations*, Clarendon Press, Oxford, 1979.
This book was followed by two others: *A Concise Guide to Semigroups and Evolution Equations*, World Scientific, Singapore, 1994 and later *Applied Nonlinear Semigroups* (in collaboration with A.C.McBride), Wiley, Chicester, 1998.
In the reviews of his books we read:
"The book is written very clearly. I am sure that many teachers in this field (including the reviewer) will always have this book in their hands. *(J. de Graaf)*.
"The book is written in a directly accessible way, also suitable for non-mathematicians" *(A.I. Filinkov)*.
"It is a merit of the authors to have combined their experiences to write a book on application of semigroups, which fills a deplorable gap in literature" *(J. Appell)*.
He came back to theoretical mechanics with his last book *Elements of Continuum Mechanics*, Carocci Ed. 2008, written in Italian in collaboration with Daniel Canarutto.
Belleni really was an applied mathematician: from complicated problems was able to extract the essential elements for appropriate mathematical models and to derive solutions in a rigorous way. At the same time, he was able to deal with mathematical problems with clarity and simplicity, making complex and extremely abstract concepts clear by means of examples and applications. Surely his degree in Physics and teaching experience in the School of Engineering contributed to create this mentality of mathematician interested in the analys of complicated problems arising from applications.
He was one of the first Italian researchers to apply methods of functional analysis and sophisticated mathematical tools for the determination of the solution in mathematical physics. Used by him,

any sophisticated and abstract mathematical technique seemed born for the purpose he pursued. Who writes can attest how the functional analysis can help in solving concrete problems and how the use of advanced mathematical techniques applied to concrete problems can lead to solving mathematical problems that would not have risen and probably not resolved only with the abstract study.

Belleni has often found himself confronted with problems not present in literature and this has always led him to 'construct' the right techniques to study the problem under examination.

Sometimes it was the generalisation of a theorem, or the alternative proof of an already known result. His scientific articles show how in mathematical physics rigor, intuition and exemplification interact in a whole.

Sometimes it was necessary to develop an instrument not yet studied: It is the case of the B-bounded Semigroups introduced and studied by him in order to face evolutionary and stationary problems with special boundary conditions. The simplicity that he used in research was also one of the characteristics of his way of life, which he was able to transmit to his students.

Two of his obituaries read:

"Our dear Aldo Belleni-Morante has gone with discretion and dignity".

"A true gentleman has gone with him".

We think that this is enough to furnish a portrait of a man who, although in his genius, was intellectually very humble. It was his habit to listen to everyone with the same attention: students, researchers, professors or scholars of clear fame.

Aldo Belleni-Morante was very happy that his teachings were valuable, constructive and effective. For this reason, we believe, how could be useful to present his unpublished research work.

Giovanni Frosali and Silvia Totaro

The Evolution of Scientific Thinking and Activity of Aldo Belleni-Morante

Belleni-Morante's first book entitled *Applied Semigroups and Evolution Equations* was published in 1979 by Oxford Clarendon Press. The book was the result of ten years of continuous research in this field. In fact, since the end of the Sixties, Belleni guessed that all evolutive problems of mathematical physics shared common features; hence some new and abstract techniques (with respect to the old ones already in use) were needed to deal with the evolutive problems which can be encountered in different/various fields of mathematical physics and engineering.

With the aim of developing a general mathematical theory for studying the evolution equations, (regardless of where they take place), it is useful to adopt a more abstract approach (as in the use of '*n*-dimensional space' for studying the solutions of a set of *n* linear equations for *n* unknowns). Starting from the concept of transition operator, which takes the system from one state to another, we are led in a natural way to the concept of a semigroup of operators. This is a family of operators satisfying various algebraic and analytic conditions reflecting the behaviour of the system.

This book gives an introduction to the theory of strongly continuous semigroups of linear operators and its application to evolution equations. Belleni's first book mainly deals with linear problems, for which the semigroup approach has been highly successful. This theory is essentially complete in the sense that it answers the basic questions of existence, uniqueness, computability and stability of solutions.

The approach followed in this book is to build a semigroup from its generator, since a strongly continuous semigroup has always a generator. In practical applications this approach is of course more important than the one based on deriving the generator from the evolution semigroup. A complete characterisation of the generators of strongly continuous semigroups is provided by a series of theorems for different types of generators, up to the Hille-Yoshida theorem, which is one of the pivotal points of the whole theory. All the theorems presented in the book are always supplemented with several examples completely worked out.

The book entitled *Applied Semigroups and Evolution Equations* appears to be self-contained since it consists of the material used by Belleni, during many years, for his course of Advanced Mechanics for students of Mathematics at the University of Florence and at the University of Bari as well as for a series of seminars held at the department of Mathematics of the Oxford University.

Prerequisites to read the book are minimal, in particular no previous knowledge of functional analysis is required.

The book *A concise guide to semigroups and evolution equations* was published in 1994 in "Series on Advances in Mathematics for Applied Sciences" Vol. 19 edited by World Scientific.
The book is based on a series of lectures held at departments of Mathematics of the University of Florence, Naples and Bari. The book is a simple and concise introduction to the semigroup theory for evolutive equations both linear and nonlinear. Since this book contains the material of a PhD course it is, necessarily, characterised by a smaller breadth of topics; nevertheless it contains a chapter devoted to olomorphic semigroup as well as a detailed example concerning heat equation. The subject is presented by a discussion of two standard boundary value problems for particle transport theory and for population theory, showing that such problems can be rewritten as evolution problems in suitable Banach spaces. Each section of the book is completed by some notes, where the relevant notions of functional analysis are explained. Some other definitions and theorems of functional analysis are discussed in the Appendix, so that the only prerequisites to read the book are classical differential and integral calculus.
If the first book is mainly devoted to linear problems or, at most, to perturbations of linear problems the second one *Applied nonlinear semigroups. An introduction* (with A.C. McBride), "Wiley Series in Mathematical Methods in Practice", 3. John Wiley & Sons, Ltd., Chichester, 1998, is devoted to nonlinear problems.
The aim of this latest book is to lead a non-expert reader to understand the technique of using the theory of semigroups of operators (both linear and nonlinear) to prove existence of solutions for evolution equations. In the nonlinear case the semigroup theory is less complete, but still provides a useful way of studying nonlinear evolution equations.
A first step towards nonlinearity is taken by examining a semilinear abstract Cauchy problem characterised by a linear term A generating a strongly continuous semigroup plus a nonlinear term F. Existence and uniqueness results of a mild solution are obtained under various conditions on F. The relevance of Lipschitz conditions, Fréchet derivatives and *a priori* bounds is shown. Several examples are studied in details in order to illustrate the theory.
The dissipative operators, to which a chapter of this book is devoted, play a pivotal role in the nonlinear theory. Accordingly, such dissipative operators are studied in some details. In particular, a chapter analyses how much of the linear theory can be extended to the nonlinear case. Firstly, semigroups generated by dissipative operators are studied, using the exponential formulae, then the theory of homogeneous abstract Cauchy problems involving nonlinear operators is developed, using a pseudo-scalar product in Banach spaces. Attention is paid both to a particular type of nonlinearity involving affine operators, which often arise in practice, and to more general perturbations of linear operators. Finally, a variety of applications of practical interest are introduced which makes the book particularly useful for researchers concerned with real life applications.

The scientific activity of Belleni is particularly characterised by novelty and originality. A first relevant example is given by B-bounded semigroups, introduced and studied by him in order to face evolution and stationary problems with particular boundary conditions. Till then, only dissipative or, at most, conservative boundary conditions were considered in transport or biomathematical problems. These problems were treated as abstract evolution problems in the framework of the theory of semigroups of operators. The situation completely changes in case of multiplying boundary conditions. Abstract theorems were found and used by Belleni and his collaborators, but this was not sufficient to fully examine the issues related to photomultipliers or to problems of biomathematics concerning growing populations.
To this regard, therefore, Belleni conceived a type of semigroup that, generated by a B operator

(bounded and with norm greater than one), was capable to lead the evolution of the problem, taking into account the multiplying effects. Knowing the form of B allows a better estimate of the number of particles or of individuals in the problem under examination. The construction is rigorous and precise and the applications confirm the utility of this new tool. Generalisations have also been performed to create non-linear "B-bounded" semigroups applied to problems with multiplying conditions of various types.

This result was made possible by applying another idea of Belleni and other collaborators: the affine semigroups. At first sight these extensions could be considered simple generalisations of some already known notions and the techniques and constructions used by Belleni may seem "simple" ones. But, deepening into a further understanding, it can be noted that, tackling concrete problems, these constructions are very much valuable. Moreover, they can be considered an easy tool to measure input and output of transport and biomathematics problems.

Simplicity is the "fil-rouge" of the construction and definitions and examples lead step by step the reader to the creation of something useful, that was just what he was looking for.

Although Belleni always thought to publish a new book concerning some peculiar aspects of semigroup theory, his last book was published in 2008 in collaboration with Daniel Canarutto and was entitled *Elementi di Meccanica dei Continui* (Carocci Editore). This last book is a concise introduction to continuous mechanics and it is, until today, the text book of the course of rational mechanics for students of the school of engineering at the University of Florence; such a course has been held for many years by Professor Belleni himself.

This educational book, although dealing with such a broad field of studies as it is the continuous mechanics (also treating, as a limit case, the behaviour of rigid body), is nevertheless written in a simple and easy way to understand.

Finally the scientific work of Aldo Belleni shows that he was one of the best applied analysts/mathematician in Italy, combining physical intuitions and interest for scientific developments in many fields with mathematical rigour.

Giovanni Frosali and Silvia Totaro

List of publications of Aldo Belleni-Morante

Books

Belleni-Morante A., *Applied semigroups and evolution equations,* (1979) Oxford Mathematical Monographs. The Clarendon Press, Oxford University Press, New York, xv+387 pp. ISBN: 0-19-853529-5.
Belleni-Morante A., *A Concise Guide to Semigroups and Evolution Equation*s, (1994) "Series on Advances in Mathematics for Applied Sciences", 19. World Scientific Publishing Co., Inc., River Edge, NJ, xiv+164 pp. ISBN: 981-02-1294-1.
Belleni-Morante A., McBride A.C., *Applied nonlinear semigroups. An introduction*, (1998) "Wiley Series in Mathematical Methods in Practice", 3. John Wiley & Sons, Ltd., Chichester, xii+275 pp. ISBN: 0-471-97867-1.
Belleni-Morante A., Canarutto D., *Elementi di Meccanica dei Continui*, (2008) Carocci Ed., pp.199 ISBN 978-88-430-4509-9.

International Journals/Proceedings/Reports

Belleni-Morante A., Monaco R., and Pieraccini S., (2009) "Photon transport in a time-dependent 3D region", *Math. Meth in Appl.* Sci, **33**, 10, 1245-1256; *Quaderni del Dipartimento di Matematica*, Politecnico di Torino, 8.
Belleni-Morante A., Lamb W. and McBride A.C., (2009) "A photon transport problem with a time-dependent point source", *Anal. Appl. (Singap.),* **7**(1), 1-19 – ISSN 0219-5305.
Belleni-Morante A., Barletti L. and Monaco R., (2007) "Kinetic models for dust coagulation in interstellar clouds", *Bull. Inst. Math. Acad. Sin.* (N.S.), **2**(2), 235-249.
Belleni-Morante A., McBride A.C and Lamb W., (2007) "Photon transport problems involving a point source", *Anal. Appl. (Singap.),* **5**(1), 77-93.
Belleni-Morante A., (2007) "Two inverse problems in photon transport theory: evaluation of a time-dependent source and of a time-dependent cross section". In: *Transport Phenomena and Kinetic Theory*, 257-271, Eds. Cercignani C. and Gabetta E., *Model. Simul. Sci. Eng. Technol.*, Birkhäuser Boston, Boston, MA.
Belleni-Morante A., Pieraccini S., Riganti R., (2007) "Numerical treatment of a time dependent inverse problem in photon transport", *Boll. Unione Mat. Ital.*, **10-B**(1), 195-211.
Belleni-Morante A., Mugelli F., (2007) "Identification of a time-dependent UV-photon source in an interstellar cloud", *Ann. Univ. Ferrara, Sez.VII, Sci. Mat.,* **53**(1), 1-11.
Belleni-Morante A., Barletti L. and Monaco R., (2004) "Kinetic models for dust coagulation in interstellar clouds", Proceedings of the 6th International Workshop on *Mathematical Aspects of Fluid and Plasma Dynamics*, Kyoto.
Belleni-Morante A., Pieraccini S., Riganti R., (2005) "Numerical treatment of a time dependent inverse problem in photon transport", *Boll. Unione Mat. Ital.*, **8-B**(3), 773-779. (Part. I).

Belleni-Morante A., Barletti L., Monaco R. and Salvarani F. (2005) "A kinetic approach to dust coagulation", *Ricerche Mat.*, **54**(1), 271-291; *Rapporto interno n. 23, Dip. di Matematica del Politecnico di Torino*, Novembre 2003.
Belleni-Morante A., Lisi M. and Totaro S., (2005) "Identification of a localized source for photon transport in a rod", *Quaderno Dipartimento di Scienze ed Informatiche" Roberto Magari"*, **468**, Siena 2005.
Belleni-Morante A., (2004) "A time dependent inverse problem in photon transport", Proceedings of the Meeting on *New Trends in Mathematical Physics*, Naples, 24-25 January 2003, Fergola P. et al. (Eds.), 1-11, World Sci. Publ., Hackensack, NJ.
Belleni-Morante A., Monaco R., Riganti R., Salvarani F., (2004) "Inverse problems in photon transport. Part I: Determination of physical and geometrical features of an interstellar cloud", *WASCOM 2003, 12th Conference on Waves and Stability in Continuous Media*, 52-59, World Sci. Publ., River Edge, NJ.
Belleni-Morante A., Riganti R., (2004) "Inverse problems in photon transport. Part II. Features of a source inside an interstellar cloud", *WASCOM 2003, 12th Conference on Waves and Stability in Continuous Media*, 60-69, World Sci. Publ., River Edge, NJ.
Belleni-Morante A., Monaco R., Riganti R., Salvarani F., (2004) "A numerical approach for inverse problems in photon transport inside an interstellar cloud", *Appl. Math. Comput.*, **154**(1), 115-126; *Rapporto interno n.26, Dipart. di Matematica del Politecnico di Torino*, dicembre 2002.
Belleni-Morante A., (2004) "Identification of the boundary surface of an interstellar cloud from a measurement of the photon far-field", *Math. Methods Appl. Sci.*, **27**(6), 627-642.
Belleni-Morante A., Barletti L., Monaco R., Salvarani F., (2003) "Kinetic approach to dust coagulation", *Quaderni del Dipartimento di Matematica Politecnico di Torino*, **23**.
Belleni-Morante A., (2003) "An inverse problem for photon transport in an interstellar cloud", *Transport Theory Statist. Phys.*, **32**(1), 73-91.
Belleni-Morante A., Mugelli F., (2002) "An inverse problem in photon transport theory: identification of the boundary surface of an interstellar cloud", *Conf. Semin. Mat. Univ. Bari*, No. **282**, pp.20.
Belleni-Morante A., (2002) "Gamma ray transport in the cardiac region: an inverse problem" (with G. Roach), *J. Math. Anal. Appl.*, **269**(1), 200-215.
Belleni-Morante A., Monaco R. and Salvarani F., (2001) An approximated solution of photon transport in a time dependent region, *Transp. Theory Stat. Phys.*, **30**, 421-438.
Belleni-Morante A., Monaco R. and Salvarani F., (2001) "Numerical simulations of a photon transport problem in a time dependent domain", *WASCOM99, 10th Conference on Waves and Stability in Continuous Media* (Vulcano), 314-324, World Sci. Publ., River Edge, NJ.
Belleni-Morante A., (2000) "A mathematical description of interstellar coagulation" (with S. Aiello et al.), *SIF Conference Proceedings*, vol. **68**, 157-162.
Belleni-Morante A., Barletti L., Cecchi-Pestellini C., Aiello S., (2000) "A Boltzmann-like model for interstellar dust coagulation", *Conf. Semin. Mat. Univ. Bari*, No. **278**, ii+ pp.19.
Belleni-Morante A., Cecchi-Pestellini C., Barletti L., Aiello S., (2000) "Mathematical methods for photon transport in random media", *J. Quantum Spectros. Radiat. Transfer*, **65**, 835-851.
Belleni-Morante A., (2000) "A mathematical model for gamma ray transport in the cardiac region" (with G. Roach), *J. Math. Anal. Appl.*, **244**, 498-514.
Belleni-Morante A., Monaco R. and Savarani F., (1999) "An approximated solution of photon transport in a time dependent region", *Quaderni del Dipartimento di Matematica Politecnico di Torino*, **9**.
Belleni-Morante A., Cecchi-Pestellini C., Barletti L., Aiello S., (1999) "Mathematical methods for photon transport in random media", Preprint n. 4810, *Harvard-Smithsonian Center for Astrophysics*.
Belleni-Morante A., Totaro S., (1999) "Photon transport in a time dependent region: a quasi-static approximation", *Conf. Sem. Mat. Univ. Bari*, No. **276**, pp. 12.
Belleni-Morante A., Cecchi-Pestellini et al., (1999) "Radiative transfer in a stochastic interstellar medium", *Transport Theory Stat. Phys.*, **28**,199-228.
Belleni-Morante A., Barletti L., (1998) "Photon transport in an interstellar cloud", Proceedings of the *IX International Conference on Waves and Stability in Continuous Media* (Bari, 1997). *Rend. Circ. Mat. Palermo* (2) Suppl., no. **57**, 39-45.

BELLENI-MORANTE A., (1998) "A nonlinear evolution problem arising from particle transport with nonhomogeneous boundary conditions", *Nonlinear Analysis*, **31**(1-2), 33-44.
BELLENI-MORANTE A., BARLETTI L., (1998) "A particle transport problem with non-homogeneous reflection boundary conditions", *Math. Methods Appl. Sci.*, **21**(11), 1049-1066; *Strathclyde Mathematics Research Report* No. 29, Sept. 1996.
BELLENI-MORANTE A., TOTARO S., (1996) "The successive reflection method in three-dimensional particle transport", *J. Math. Phys.*, **37**(6), 2815-2823.
BELLENI-MORANTE A., (1996) "On some properties of B-quasi bounded semigroups", *Quaderni Dipartimento Ingegneria Civile Fir. Sezione Modelli Matematici*, **1/96**, Firenze.
BELLENI-MORANTE A., LAURO G., MONACO R., (1996) "Modelling and numerical simulation of contamination processes in random media", *Maths Comput. in Simulat.*, **42**, 85-95.
BELLENI-MORANTE A., BUSONI G., (1997) "A three dimensional Boltzmann-like model of outgassing and contamination", *European J. Appl. Math.*, **8**(2), 229-249; *Quaderni Dipartimento Ingegneria Civile Firenze, Sez. Modelli Matematici*, **3/93**, Firenze (1993).
BELLENI-MORANTE A., (1996) B-bounded semigroups and applications, *Ann. Mat. Pura e Appl.*, **170**(4), 359-376.
BELLENI-MORANTE A., SACCOMANDI G., (1995) "Time dependent photon transport in a three dimensional interstellar cloud with stochastic clumps", *Astrophysics and Space Science*, **234**, 85-105.
BELLENI-MORANTE A., TOTARO S., (1995) "The free streaming operator: a mathematical model for diffusive multiplying boundary conditions", *Math. Methods Appl. Sci.*, **18**(14), 1165-1180.
BELLENI-MORANTE A., (1994) "The multiple collision method revisited", *Quaderni Dipartimento Ingegneria Civile Fir. Sezione Modelli Matematici*, **4/94**, Firenze.
BELLENI-MORANTE A., MORO A., AIELLO S., CECCHI-PESTELLINI C., (1994) "Photon transport in an interstellar cloud with stochastic clumps: the three-dimensional case", *Confer. Sem. Mat. Univ. Bari*, No. **255**, pp.22.
BELLENI-MORANTE A., TOTARO S., (1994) "Particle transport: the successive reflection method" (with S. Totaro), *Quaderni Dipartimento Ingegneria Civile Firenze, Sez. Modelli Matematici*, **3/94**, Firenze.
BELLENI-MORANTE A., MORO A., (1996) "Time dependent photon transport in an interstellar cloud with stochastic clumps" (with A. Moro), Transport Theory Statist. Phys. **25**(1), 85-101, *Quaderni Dipartimento Ingegneria Civile Firenze Sez. Modelli Matematici*, **2/93**, Firenze (1993).
BELLENI-MORANTE A., BUSONI G., (1993) "Some remarks on densely defined streaming operators", Math. Comput. Modelling, **21**(8) 13-19 (1995), *Quaderni Dipartimento Ingegneria Civile Firenze Sez. Modelli Matematici*, **1/93**, Firenze.
BELLENI-MORANTE A., BUSONI G., (1993) "Outgassing and contamination: strict solution of a Boltzmann-like model", *Math. Methods Appl. Sci.*, **16**, 799-817.
BELLENI-MORANTE A., MORO A., AIELLO S., CECCHI-PESTELLINI C., (1992) "A simple mathematical model for photon transport in an interstellar cloud with stochastic clumps", *Quaderni Dipartimento Ingegneria Civile Firenze Sez. Modelli Matematici*, **3/92**, Firenze.
BELLENI-MORANTE A., TOTARO S., (1992) "A mathematical model for transport and absorption of particles of a poison gas", *Confer. Sem. Mat. Univ. Bari*, No. **247**, pp.21.
BELLENI-MORANTE A., TOTARO S., (1992) "Nonlinear transport and absorption of particles of a poison gas", *Quaderni Dipartimento Ingegneria Civile Firenze Sez. Modelli Matematici*, **1/92**, Firenze.
BELLENI-MORANTE A., (1992) "A mathematical model for particle transport with mollified boundary conditions", *Math. Comput. Modelling*, **16**(3),131-137.
BELLENI-MORANTE A., FASANO A., Primicerio M., (1992) "Existence and uniqueness of the solution of a problem of outgassing and contamination", *Adv. Math. Sci Appl.*, **1**(2), 195-206.
BELLENI-MORANTE A., TINÉ S., AIELLO S., CECCHI PESTELLINI C., (1992) "A solution to the problem of radiation transfer in inhomogeneous media by the SHM", *J. Quantum Spectr. and Radiation Transfer*, **47**, 95-102.
BELLENI-MORANTE A., (1991) "The free streaming operator in asymmetric Sobolev spaces", *Math. Models Methods Appl. Sci.*, **1**(4), 421-436.
BELLENI-MORANTE A., LAURO G., (1991) "On some spectral properties of the streaming operator with mol-

lified boundary conditions, Semigroup theory and evolution equations" (Delft, 1989), 311-324, *Lecture Notes in Pure and Appl.Math.*, **135**, Dekker, New York.
BELLENI-MORANTE A., (1991) "Particle streaming in asymmetric Sobolev spaces", *Quaderni Dipartimento Ingegneria Civile, Firenze, Sez. Modelli Matematici*, **1/91**, Firenze.
BELLENI-MORANTE A., TOTARO S., (1990) "A mathematical model for human body temperature" (with S.Totaro), *Conf. Sem. Mat. Univ. Bari*, No. **235**, pp.26.
BELLENI-MORANTE A., FASANO A., Primicerio M., (1990) "Outgassing and mass exchange between two surfaces", Proceedings of the Fifth European Conference on *Mathematics in Industry* (Lahti, 1990), 143-146, *European Consort. Math. Indust.*, **7**, Teubner, Stuttgart.
BELLENI-MORANTE A., (1990) "A mathematical model of brain injuries due to angular accelerations. Memoirs of mathematical physics in honor of Giovanni Carini on his 70th birthday (Italian)", *Atti Accad. Peloritana Pericolanti Cl. Sci. Fis. Mat. Natur.*, **68**, suppl. 1, 381- 402.
BELLENI-MORANTE A., (1990) "A mathematical model for brain injuries", *Quaderni Dipartimento Ingegneria Civile, Firenze, Sez. Modelli Matematici*, **4/90**, Firenze.
BELLENI-MORANTE A., (1990) On some spectral properties of the streaming operator with mollified boundary conditions, *Quaderni Dipartimento Ingegneria Civile, Firenze, Sez. Modelli Matematici*, **3/90**.
BELLENI-MORANTE A., TOTARO S., (1990) "The discrete ordinate method for particle transport problems with Maxwell type boundary conditions", *Quaderni Dipartimento Ingegneria Civile, Firenze, Sez. Modelli Matematici*, **2/90**.
BELLENI-MORANTE A., TOTARO S., (1990) "An upper bound for the solution of a nonlinear particle transport problem", *Quaderni Dipartimento Ingegneria Civile, Firenze, Sez. Modelli Matematici*, **1/90**.
BELLENI-MORANTE A., TOTARO S., (1990) "The discrete ordinate method for particle transport problems", *Atti del X Convegno AIMETA*, vol.I, 3-6, Pisa.
BELLENI-MORANTE A., LAURO G., (1990) "The kinetics equations on a multiplying medium with free boundaries", *Math. Methods Appl.Sci.*, **13**(3), 263-272.
BELLENI-MORANTE A., TOTARO S., (1989) "On the discrete ordinate method in particle transport", *Rend. Mat. Appl.*, Serie VII Vol. **9**(3), 383-399.
BELLENI-MORANTE A., TOTARO S., (1989) "A mathematical model for human body temperature", *Quaderni Dipartimento Ingegneria Civile, Firenze, Sez. Modelli Matematici*, **2/89**.
BELLENI-MORANTE A., LAURO G., (1989) "Neutron transport and heat transfer in a region with free boundaries", *Conf. Sem. Math. Univ. Bari*, No. **230**, Laterza, Bari, pp.39.
BELLENI-MORANTE A., (1998) "A mathematical model for particle transport with mollified boundary conditions", *Quaderni Dipartimento Ingegneria Civile, Firenze, Sez. Modelli Matematici*, **1/89**.
BELLENI-MORANTE A., LAURO G., (1988) "An evolution problem for neutron transport and heat transfer in a slab with moving boundaries", *III Internat. Workshop on Plasma and Fluid Dynamics*, Salice Terme.
BELLENI-MORANTE A., LAURO G., (1988) "A simple model for a slab reactor with moving boundaries", *Transp. Theory Stat. Phys.*, **17**, 225-239.
BELLENI-MORANTE A., (1979) "A problem of evolution in a time dependent domain", *Conf. Seminario di Matematica Bari*, n. **169**, 1-18.
BELLENI-MORANTE A., BUSONI G., and FROSALI G., (1979) "A nonlinear problem of evolution arising from radiation transport theory", *Nonlinear Analysis*, **3**(6), 755-772.
BELLENI-MORANTE A., (1979) "Particle transport in a slab bounded by capturing and reflecting planes", *Riv. Mat. Univ. Parma*, **4**(5), Part 1, 85-98.
BELLENI-MORANTE A., BARONE E., (1978) "A nonlinear initial value problem arising from kinetic theory of vehicular traffic", *Transport Theory Statist. Phys.*, **7**(1-2), 61-79.
BELLENI-MORANTE A., (1978) "A problem of evolution arising from particle transport with quadratic nonlinearities", *Confer. Sem. Mat. Univ. Bari*, n.**155**, pp.28.
BELLENI-MORANTE A., (1978) "An integro-differential equation arising from the theory of heat conduction in rigid materials with memory", *Boll. Un. Mat. Ital. B (5)*, **15-B**(2), 470-482.
BELLENI-MORANTE A., (1977) "Global solution for a problem of neutron transport with temperature feedback". In: *Nonlinear systems and applications (Proc. Internat. Conf., Univ. Texas, Arlington, Tex., 1976)*, Ed. By V. Lakshmikanntham, pp. 413-424, Academic Press, New York.

BELLENI-MORANTE A., FROSALI G., (1977) "Global solution of a nonlinear initial-value problem of vehicular traffic", *Boll. Un. Mat. Ital. A (5)*, **14-A**(1), 71-81.
BELLENI-MORANTE A., BUSONI G. and CAPASSO V., (1976/77) "Global solution of a nonlinear neutron transport problem with temperature feedback", *Nonlinear Anal.*, **1**(6), 651-665.
BELLENI-MORANTE A., (1976) "Neutron transport with temperature feedback", *Nuclear Science Eng.*, **59**, 56-58.
BELLENI-MORANTE A., FARANO R., (1976) "Neutron transport in a slab with moving boundaries", *SIAM J. Appl. Math.*, **3**(4), 591-599.
BELLENI-MORANTE A., (1976) "Trasporto di neutroni con sezioni d'urto dipendenti dalla temperatura", A*nn. Mat. Pura Appl. (4)*, **108**, 85-95.
BELLENI-MORANTE A., (1976) "A rigorous derivation of the reactor kinetics equations from the Chapman-Kolmogorov system", *Riv. Mat. Univ. Parma (4)*, **2**, 239-250.
BELLENI-MORANTE A., (1976) "Erratum: Nonlinear kinetic theory of vehicular traffic", *J. Math. Anal. Appl.*, **53**(1), 186.
BELLENI-MORANTE A., RADAELLI A., (1975) "A stochastic model of vehicular traffic", *Matematiche (Catania)*, **XXX**(1), 1-14.
BELLENI-MORANTE A., (1975) "On Chapman-Kolmogorov equations for a neutron population in a multiplying assembly", *J. Math. Phys.*, **16**, 585-589.
BELLENI-MORANTE A., (1975) "On the stability of Couette-Benard M.H.D. flows of binary mixtures", *Ann. Mat. Pura Appl. (4)*, **106**, 353-374.
BELLENI-MORANTE A., (1974) "Nonlinear kinetic theory of vehicular traffic" (with E. Barone), J. Math. Anal. Appl., **47**, 443-457.
BELLENI-MORANTE A., PAGLIARINI A., (1974) "Two-group kinetic theory of vehicular traffic", *Meccanica,* **9** (3), 151-156.
BELLENI-MORANTE A., VITOCOLONNA C., (1974) "Discretization of the time variable in evolution equations", *J. Inst. Math. Applic.*, **14**, 105-112.
BELLENI-MORANTE A., (1973) "Stability in kinetic theory of vehicular traffic", *Meccanica*, **8**, 11-15.
BELLENI-MORANTE A., (1972) "Multigroup neutron transport" (with G. Busoni), *J. Math. Phys.*, **13**, 1146-1150.
BELLENI-MORANTE A., (1971) "The discretization of the neutron transport equation", *Proc. of the Topical Meeting on New Developments in Reactor Mathematics ad Applications*, Idaho Falls, March 1971.
BELLENI-MORANTE A., (1971) "Neutron transport and kinetics in a reactor composed of two loosely coupled cores", *Proc. of the National Topical Meeting on Coupled Reactor Kinetics*, College Station, Texas.
BELLENI-MORANTE A., (1970) "Neutron transport in a nonuniform slab with generalized boundary conditions", J. *Math. Phys.*, **11**(5), 1553-1558.
BELLENI-MORANTE A., (1970) "The initial value problem for neutron transport in a slab with perfect reflection boundary conditions", *J. Math. Anal. Appl.*, **30**(2), 353-374.
BELLENI-MORANTE A., (1970) "Neutron transport in a nonuniform slab with delayed neutron production", *Matematiche*, XXV, 69-83.
BELLENI-MORANTE A., (1970) "The initial-value problem for neutron transport in a finite body with generalized boundary conditions", *Meccanica*, **5**, 253-261.
BELLENI-MORANTE A., BUSONI G., MOTTA M., (1969) "Neutron leakage delay-time distribution in reactor kinetics", *Energia Nucleare*, **16**, 240.
BELLENI-MORANTE A., (1969) "The initial-value problem for neutron transport in a coupled core assembly", *Meccanica*, **4**, 1-7.
BELLENI-MORANTE A., (1969) "Un problema non lineare di trasporto di neutroni" (Italian), Ann. *Mat. Pura Appl. (4)*, **82**, 97-114.
BELLENI-MORANTE A., (1968). "On the principle of gyroscopic effect for self-excited rigid bodies", *Meccanica*, **3**, 106, 867-870.
BELLENI-MORANTE A., (1968) "Un'osservazione sopra una classe di sollecitazioni G-autonome", *Riv. Mat. Univ. Parma*, **9**(2), 275.
BELLENI-MORANTE A., (1968) "Un problema di convezione forzata e di conduzione del calore con sorgenti dipendenti dalla temperatura", *Ann. Mat. Pura Appl.*, **78**(1), 69-90.

Belleni-Morante A., (1967) "Sul principio dell'effetto giroscopico", *Rend. Sem. Mat. Univ. Padova*, **39**, 304.
Belleni-Morante A., (1967) "On the delay time distribution function in coupled reactor kinetics", *Nukleonik*, **10**(4) 217-223 (1967).
Belleni-Morante A., (1967) "Effect of first-flight neutrons on coupled reactor kinetic behaviour", *J. Nuclear Energy*, 21, 867-870.
Belleni-Morante A., (1966) "Diffusione di neutroni in un mezzo costituito da due regioni debolmente accoppiate", *Rend. Acc. Naz. Lincei (8)*, **11**, 80.
Belleni-Morante A., (1966) "The kinetic behaviour of a reactor composed of two loosely coupled cores with time-dependent coupling coefficients", *Nukleonik*, **8**, 291-296.
Belleni-Morante A., (1966) "Neutron transport and reactor kinetics", *Energia Nucleare*, **13**, 304-311.
Belleni-Morante A., (1966) "The reactor kinetics equation as a direct consequence of the Boltzmann equation", *Nukleonik*, **8**(1) 33-37.
Belleni-Morante A., (1965) "L'equazione della cinetica dei reattori nucleari" (Italian), *Boll. Un. Mat. Ital. (3)*, **20**, 486-497.
Belleni-Morante A., (1965) "Su un problema cilindrico di convezione forzata e di conduzione del calore in un reattore nucleare", B*oll. Un. Mat. Ital.* (3), **20**, 106-113.
Belleni-Morante A., (1964) "Esistenza ed unicità della soluzione di un problema cilindrico di convezione forzata e di conduzione del calore" (Italian), *Atti Sem. Mat. Fis. Univ. Modena*, **13**, 62-73.
Belleni-Morante A., (1964) "The kinetic behaviour of a reactor composed of G loosely coupled cores: integral formulation", J. *Nuclear Energy*, **18**, 547-559.
Belleni-Morante A., (1964) "A heat transfer problem in a hollow cylindrical cell", *Energia Nucleare"*, **11**, 134-136.
Belleni-Morante A., (1964) "On the best mean decay constant for reactor kinetics equations with one group of delayed neutrons", *Nukleonik*, **6**(4) 209-211.
Belleni-Morante A., (1964) "Solution of the nuclear reactor kinetics equations by a method of successive approximations", *J. Nucl. Energy*, **6**, 209-211.
Belleni-Morante A., (1963) "On some properties of the reactor kinetics integral equation: the perturbed kernel method", *Nukleonik*, **5**, 293.
Belleni-Morante A., (1963) "Solution of the nuclear reactor kinetics equations by a method of successive approximations", *J. Nuclear Energy*, **17**, 439.
Belleni-Morante A., (1963) "The integral formulation of the reactor kinetics equations", *Nukleonik*, **5**, 183.
Belleni-Morante A., (1962) "Influenza della temperatura sulla diffusione dei neutroni nei reattori nucleari omogenei" (Italian), *Riv. Mat. Univ. Parma (2)*, **3**, 309-324.
Belleni-Morante A., (1961) "Su un problema sferico di diffusione neutronica" (Italian), *Riv. Mat. Univ. Parma (2)*, **2**, 53-59.

International Journals honoured Aldo Belleni-Morante

Mathemathical Methods in the Applied Sciences, vol. **33**, 10 (2010) Wiley-Blackwell.
Physica D, Nonlinear Phenomena, Evolution Equations in Pure and Applied Sciences, vol. **239**, 15, (2010) Elsevier.

The Manuscript

SEMIGROUPS AND EVOLUTION EQUATIONS IN LOCALLY CONVEX SPACES : AN INTRODUCTION

OR

APPLIED SEMIGROUPS IN LOCALLY CONVEX SPACES

ABM , AMcB , WL

CONTENTS

APPENDIX A (to Chapter 0.)

A.1 NORMED SPACES, BANACH SPACES

A.2 BOUNDED OPERATORS, CLOSED OPERATORS

A.3 SEMIGROUPS AND INITIAL VALUE PROBLEMS

A.3.1 SEMIGROUPS GENERATED BY $B \in \mathcal{B}(X)$

A.3.2 SEMIGROUPS GENERATED BY $A \in \mathcal{C}(X)$

A.3.3. PERTURBATIONS OF THE GENERATOR, "SUHADOLC-VIDAV" PERTURBATIONS

A.3.4 HOMOGENEOUS AND NON HOMOGENEOUS INITIAL VALUE PROBLEMS

A.3.5 THE SEMILINEAR PROBLEM

A.36 DUAL SEMIGROUPS, SUN-DUAL SEMIGROUPS

6/11/08
→ 9/12/08

1. LOCALLY CONVEX SPACES

1.1. TOPOLOGICAL LINEAR SPACES

A nonempty set X is a real vector space (or a real linear space) if, given any $f \in X$, $g \in X$ and $\alpha \in \mathbb{R}$ (α is a dimensionless real number), the operations $f+g$ and αf are defined with $f+g \in X$, $\alpha f \in X$, and satisfy the axioms of elementary algebra.

Assume that the family $\mathcal{F} = \{ G_\lambda, \lambda \in \Lambda \}$ of subset of X satisfies the following assumptions

ass. 1) X and the void set $\emptyset$ belong to $\mathcal{F}$;

ass. 2) any union of elements of $\mathcal{F}$ still belongs to $\mathcal{F}$:

$$\bigcup_\lambda G_\lambda \in \mathcal{F} \quad , \quad G_\lambda \in \mathcal{F} \; ;$$

ass. 3) the intersection of any finite number of elements of $\mathcal{F}$ still belongs to $\mathcal{F}$:

$$\bigcap_{i=1}^{n} G_i \in \mathcal{F} \quad , \quad G_i \in \mathcal{F} \; , \; i = 1, 2, \dots, n \; .$$

Then, we say that the G_λ's are "open subsets" of X and X becomes a (linear) topological space, with topology defined by $\mathcal{F}$.

Remark 1.1. The family of open sets $\mathcal{F}$ defines a topology in X in the following sense. The element $f \in X$ is "close" to a given $f_0 \in X$ if a suitable G_{λ_0}, $\lambda_0 \in \Lambda$ exists, such that $f_0 \in G_{\lambda_0}$ and $f \in G_{\lambda_0}$. ■

Remark 1.2. Normed spaces (with norm $\|\cdot\|$) are a particular case of linear topological spaces. A subset G of a normed space X is "open" if, given any $f \in X$, a suitable $\varepsilon > 0$ can be found such that the set $\{g : \|g - f\| < \varepsilon\}$ is contained in G. ■

In what follows, we shall consider only topological spaces X that satisfy Hausdorff's axiom of separation:

ass. 4) given $f \in X$ and $g \in X$, with $f \neq g$, two open sets $G_f \in \mathcal{T}$ and $G_g \in \mathcal{T}$ can be found, such that $f \in G_f$, $g \in G_g$ with $G_f \cap G_g = \emptyset$.

1.2. SEMINORMS

Let X be a linear space and assume that $p(f)$, $f \in X$, is a real valued function, defined on the whole X. We say that p is a seminorm if

s1) $p(f+g) \leq p(f) + p(g)$ $\forall f, g \in X$ ("triangle inequality")

s2) $p(\alpha f) = |\alpha| p(f)$, $\forall f \in X$, $\alpha \in \mathbb{R}$.

Conditions s1) and s2) imply that p also has the following properties

σ1) $p(\theta) = 0$ where θ is the zero element of X,

σ2) $p(f-g) \geq |p(f) - p(g)|$ $\forall f, g \in X$,

σ3) $p(f) \geq 0$ $\forall f \in X$.

In fact, s2) gives

$p(\theta) = p(0 f) = 0\, p(f) = 0$.

Furthermore, by using s1) and s2) we obtain

2.6 THE NON HOMOGENEOUS INITIAL VALUE PROBLEM

The following non homogeneous initial value problem in the locally convex space X is of interest for applications:

$$\frac{\partial u(t)}{\partial t} = A\,u(t) + q(t),\ t>0;\quad u(0)=u_0, \tag{2.39}$$

see Theorem 2.15. In (2.39), the term $q(t)$ may model a source of particles (photons, electrons, ...), of thermal energy, or an immigration source (if $u(t)=u(\cdot,t)$ is a population density), and so on.

We shall first consider the case in which A is a continuous linear operator, see Section 1.5.

Theorem 2.18.

Assume that the continuous linear operator A is the generator of the equicontinuous semigroup $\{T(t)=\exp(tA),\ t\geq 0\}$, (see Theorem 2.8), and that $q\in C^1([0,\bar t],X)$, where $\bar t$ is arbitrarily assigned. Then, the unique (strong) solution of (2.39) is given by

$$u(t) = T(t)u_0 + \int_0^t T(t-s)q(s)\,ds \tag{2.40}$$

$\forall u_0 \in X$.

Proof.

If we let

$$u(t) = T(t)u_0 + v(t)$$

with

$$v(t) = \int_0^t T(t-s)q(s)\,ds = \int_0^t T(s)q(t-s)\,ds,$$

the proof is similar to that used when X is a Banach space (see ABH, AN-B, Th 2.45). ∎

Or for on the case in which A is not necessarily continuous but is the generator of a quasi-equicontinuous semigroup and $q \in C^1([0,\bar{t}], X)$ .... ??? (At the end of his 1985 paper, Y.H. Choe writes that he was going to publish something on this problem ...)

Khan, Wilson, any news ?

Example 2.19

Let X be the space of Example 2.13 and assume that $q(t) = q(\cdot, t)$, where $q(x,t) = q_0(x)\varphi(t)$ with $q_0 \in L^1(-a,a)$ and $\varphi \in C^1([0,\bar{t}])$. We have

$$\frac{q(t+\theta)-q(t)}{h} - q_0\varphi'(t) = q_0\left[\frac{\varphi(t+\theta)-\varphi(t)}{h} - \varphi'(t)\right] ;$$

hence, given any q_m and $\varepsilon > 0$, we obtain

$$q_m\left(\frac{q(t+\theta)-q(t)}{h} - q_0\varphi'(t)\right) \le \left|\frac{\varphi(t+\theta)-\varphi(t)}{h} - \varphi'(t)\right| \int_{-q_m}^{q_m} |q_0(x)|\,dx$$

$$\le \varepsilon \int_{-a}^{+a} |q_0(x)|\,dx = \varepsilon \|q_0\|_1 \quad , \quad \forall\, |\theta| < h_0 = h_0(\varepsilon).$$

Thus, $dq/dt = q_0\varphi'$ (and dq/dt is continuous). ∎